Microwaves

Series 4

Problem 1

The cutoff frequency of a waveguide mode is of 2 GHz, and the frequency of the signal is of 3 GHz. Give the wavelength, the phase and group velocities and the wave impedance for

a) a TE mode
b) a TM node

The wavelength, phase velocity and group velocity are the same for TE and TM modes. They are given by :

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} \quad v_\varphi = \frac{1}{\sqrt{\varepsilon\mu}} \frac{1}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} \quad v_g = \frac{1}{\sqrt{\varepsilon\mu}} \sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

The wave impedances are different ::

$$Z_{TE} = \sqrt{\frac{\mu}{\varepsilon}} \frac{1}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} \quad Z_{TM} = \sqrt{\frac{\mu}{\varepsilon}} \sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

The guide is filled with air, thus

$$\frac{1}{\sqrt{\varepsilon\mu}} = c = 3 \ 10^8 \ m/s \quad \sqrt{\frac{\mu}{\varepsilon}} = 120\pi \quad \Omega$$

and we finally get:

$$\lambda_g = 13.42 \ cm \quad v_\varphi = 4.02 \ 10^8 \ m/s \quad v_g = 2.24 \ 10^8 \ m/s$$

$$Z_{TE} = 504.4 \ \Omega \quad Z_{TM} = 280.8 \ \Omega$$

Problem 2

Consider two waveguides, the first of section 20mm x 12 mm and the second of section 16mm x 12mm.

a) what are the cutoff frequencies of the dominant mode for these waveguides ?
b) what is the first higher order mode and what is its cutoff frequency for both waveguides ?
c) We connect the two guides together. What will be the reflection coefficient at the interface for a signal having a frequency of 10 GHz ?

For both waveguides, the dominant mode is the TE_{10} modes. Its cutoff frequency is given by :

$$f_c = \frac{\omega_c}{2\pi} = \frac{c}{2a} = \begin{cases} 7.5 \ GHz \ (guide \ 1) \\ 9.375 \ GHz \ (guide \ 2) \end{cases}$$

For both guides, the first higher order mode is the TE_{01} mode. Its cutoff frequency is given by :

$$f_c = \frac{\omega_c}{2\pi} = \frac{c}{2b} = 12.5 \ GHz$$

At 10 GHz, only the dominant mode is able to propagate in these guides. The reflection coefficient is given by :

$$\rho = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

where Z_1 is the wave impedance of the dominant mode in the first guide and Z_2 is the wave impedance of the dominant mode in the second guide.

$$Z = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} = \frac{120\pi}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \Rightarrow \begin{cases} Z_1 = 570 \ \Omega \\ Z_2 = 1083 \ \Omega \end{cases}$$

$$\rho = 0.31$$

Problem 3

We use a square waveguide filled with air, having a side dimension of 5 cm, in order to transmit a signal at a frequency of 6.2 GHz. How many TE modes can propagate at this frequency ? For each mode, what is the time taken for a modulated pulse to travel through a guide 100m long ?

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fc (TE10) = fc(TE01)=3 GHz propagating modes

fc(TE11)=4.243 GHz propagating mode

fc(TE20)=fc(TE02)=6 GHz propagating modes

fc(TE21)=fc(TE12)=6.708 GHz (not propagating)

There are thus only five propagating modes TE.

To find the propagation time of a modulated impulsion, we need the group velocity :

$v_{\mathrm{g}} = \mathrm{c}_0\sqrt{1-\left(p\mathrm{c}_0/\omega\right)^2} = \mathrm{c}_0\sqrt{1-\left(f_{\mathrm{c}}/f\right)^2}$ and the time is given by :

$$t = \frac{l}{v_g} = \frac{l}{\mathrm{c}_0\sqrt{1-\left(f_{\mathrm{c}}/f\right)^2}} = \frac{100}{3\cdot 10^8\sqrt{1-\left(f_{\mathrm{c}}/6\right)^2}} = \frac{33,333\cdot 10^{-8}}{\sqrt{1-\left(f_{\mathrm{c}}/6.2\right)^2}}\,\mathrm{s} = \frac{333,33}{\sqrt{1-\left(f_{\mathrm{c}}/6.2\right)^2}}\,\mathrm{ns}$$

For the five modes, we get ::

$$t_{10}^{\mathrm{TE}} = t_{10}^{\mathrm{TE}} = \frac{333,33}{\sqrt{1-\left(3/6.2\right)^2}} = 380.89\,\mathrm{ns}$$

$$t_{11}^{\mathrm{TE}} = \frac{333,33}{\sqrt{1-\left(4,24/6\right)^2}} = 456.86\,\mathrm{ns}$$

$$t_{20}^{\mathrm{TE}} = t_{02}^{\mathrm{TE}} = \frac{333,33}{\sqrt{1-\left(6/6.2\right)^2}} = 1323\,\mathrm{ns}$$

We note that the travel time is very different from mode to mode, the signal becoming slower as we get nearer cutoff.